Permeability Prediction for Reservoir Sandstones and Basement Rocks Based on Fractal Pore Space Geometry

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Summary

Estimating permeability from grain-size distributions or from well logs is attractive but difficult. In this paper we present a new, generally applicable, and relatively inexpensive approach which yields information on permeability on the core sample and on the borehole scale. The approach is theoretically based on a fractal model for the internal structure of a porous medium. It yields a general and petrophysically justified relation linking porosity to permeability, which may be calculated either from porosity or from the pore radius distribution. This general relation can be tuned to the entire spectrum of sandstones ranging from clean to shaly sandstones. The resulting expressions for the different rock types are calibrated to a comprehensive data set of petrophysical and petrographical rock properties measured on 640 sandstone core samples of the Rotliegend (Lower Permian) in northeast Germany. Permeability calculated with this procedure from industry porosity logs compares very well with permeability measured on sedimentary and metamorphic rock samples.

Introduction

Permeability is one of the key petrophysical parameters for the management of hydrocarbon and geothermal reservoirs as well as for aquifers. However, its magnitude may vary over several orders of magnitude, even for a single rock type such as sandstone. Moreover, permeability is very sensitive to changes in overburden pressure or to diagenetic alterations. For instance, drastic permeability reductions result from the growth of minute amounts of secondary clay minerals on quartz grains, since this changes the geometry of the hydraulic capillaries. Generally speaking, permeability $k$ is a function of the properties of the pore space, such as porosity $\phi$ and several structural parameters. In the past, different empirical approaches were used to describe the observed highly nonlinear dependence of permeability on porosity by exponential or power-law relationships. However, these purely empirical approaches lack a petrophysical motivation. The slopes of different linear regressions vary from moderate for sandy clay and silt to very steep for channel sandstones. This suggests that it would be very difficult to explain the $\phi$-$k$ relationships of different lithologies with one single empirical expression. As a consequence, a log($\phi$)-log($k$) plot of data from different sandstones with various lithologies is only very weakly correlated.

Most current models express $k$ as the product of $\phi$ and a size parameter, taken to different powers. This size parameter may be either a grain diameter, a pore radius, or a specific surface. In these power laws, the exponent of the size parameter (or its reciprocal) is equal or close to 2 while the exponent of $\phi$ in the nominator usually varies between 1 and 7, and $(1-\phi)^2$ appears in the denominator. The petrophysical base of most of the non-empirical models is the Kozeny-Carman equation which links permeability to the effective pore radius and the formation factor $F$. However, none of these simple models can explain the strong dependence of permeability on porosity observed, for instance, in channel sandstones. There the pores are compressed to such a degree that the hydraulic path is reduced to the free space between adjacent grain asperities. Thus, the simple model of unconsolidated sand packing no longer applies, in which a combination of ideal, smooth spheres and large voids in between is assumed. This illustrates that a real-world pore system is more complicated. It needs to be viewed rather as a superposition of distinct structures at different scales than as a simple arrangement of spheres or tubes.

In basin analysis it is common practice to use simple relationships of the modified Kozeny-Carman type. These expressions yield bad permeability predictions, if the size parameter is treated as a constant for lack of further information.
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In contrast to other methods and to strictly empirical relations, fractal theory combined with petrophysical relations provides alternative approaches. Number and definition of shape parameters required by different models vary according to the approach chosen. A model which is particularly close to the natural appearance of pore space in sedimentary rocks is the so-called "pigeon hole" model. It is based on data measured on core samples from a variety of hydrocarbon reservoir rocks in northwest Germany. It yields petrophysically justified relations between various geometric, storage, and transport parameters of these reservoir rocks. Like other fractal models, it is based on the observation that the shape of the inner surface of rock pores follows a self-similar rule. Thus, the theory of fractals can be applied. A key parameter in this theory is the fractal dimension D.

We apply this approach to a petrophysical data set which allows a very thorough characterization of Rotliegend (Lower Permian) sandstone samples. It had been compiled over a period of 15 years in East German hydrocarbon industry laboratories. It comprises of measured porosity and permeability as well as of distributions of pore radii and grain sizes. A comparison of samples with equal porosity but different permeability reveals a strong dependence of permeability on the distribution of pore radii. Also, the clay mineral content appears to be an important parameter. We use this comprehensive data set to calibrate the general fractal permeability-porosity relationship. The result is a simple relation linking permeability either to laboratory derived pore radius distributions or to porosity obtained from industry well logs.

Theory

A fundamental expression for the permeability k of a porous medium is given by the modified Kozeny-Carman equation:

\[ k = \frac{r_{eff}^2}{8F} \]  

(1)

where \( F \) is the formation factor, which is defined as the ratio of tortuosity \( T \) and porosity \( \phi \):

\[ F = \frac{T}{\phi} \]  

(2)

The formation (resistivity) factor is originally defined as the ratio of the resistivity of a porous medium saturated with an electrolyte and the resistivity of this electrolyte itself. It is a purely geometric parameter, which describes the way in which the porous medium obstructs transport processes, such as the passage of ions or even uncharged molecules. It is related to permeability via the Kozeny-Carman equation (1). Pape et al. (1987a) showed that tortuosity behaves as a fractal and depends on the ratio of \( r_{eff} \) and \( r_{grain} \) (Figure 1) with an exponent involving the fractal dimension \( D \):

\[ T = 1.34 \left( \frac{r_{grain}}{r_{eff}} \right)^{0.67(D-2)}. \]  

(3)

This implies that tortuosity increases with increasing fractal dimension. However, this relation is valid only in the range of 2 < \( D < 2.4 \). Strongly fractured rocks, such as mylonites or claystones, show fractal dimensions in the range of 2.4 < \( D < 3 \). These rocks are characterized by a high degree of connectivity of the pore system which, in turn, reduces the tortuosity. An empirical relationship between \( F \) and \( \phi \) is expressed by Archie’s first law:

\[ F = a\phi^m \]  

(4)

Here \( a \) is a factor depending on lithology, and \( m \) is the cementation or tortuosity factor which depends on rock structure. The parameters \( a \) and \( m \) vary in the ranges 0.6 < \( a < 2 \), and 1 < \( m < 3 \). Later, Archie’s law (equation 4) was also formally derived from a fractal model for porous
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rocks by Pape and Schopper (1988) and Shashwati and Tarafdar (1997). For instance, for a pigeon hole model of common sandstones (Figure 1) this derivation starts from equation (2), which expresses $F$ by using tortuosity $T$ and porosity $\phi$, and the following equations (Pape et al., 1987a) which express the fractal relationship between $T$ and $\phi$:

$$T = 1.34 \left( \frac{r_{\text{grain}}}{r_{\text{eff}}} \right)^{0.67} \left( \frac{r_{\text{eff}}}{r_{\text{grain}}} \right)^{0.24} = 1.34 \left( \frac{r_{\text{grain}}}{r_{\text{eff}}} \right)^{0.67} \left( \frac{r_{\text{eff}}}{r_{\text{grain}}} \right)^{0.24} \tag{5}$$

$$\phi = 0.534 \left( \frac{r_{\text{grain}}}{r_{\text{eff}}} \right)^{0.39} \left( \frac{r_{\text{eff}}}{r_{\text{grain}}} \right)^{0.25} \tag{6}$$

with $D=2.36$. Neglecting slight differences in the exponents, equations (5) and (6) for tortuosity and porosity yield

$$T = 0.67 \phi \tag{7}$$

and

$$r_{\text{eff}}^2 = r_{\text{grain}}^2 (2\phi)^{8} \tag{8}$$

Using the Kozeny-Carman equation (1), the formation factor, equation (2), tortuosity, equation (7), the effective radius, equation (8) with the default value $r_{\text{grain}} = 200 000$ nm (an average grain radius in our sandstone data sets), $k$ can now be written as

$$k = 191 (10 \phi)^{10} \quad \text{(in nm}^2\text{)}. \tag{9}$$

Equation 9 for permeability defines a straight line asymptote in a log-log plot of $k$ versus $\phi$. It is valid for porosities greater than 0.1, whereas for lower porosities the measured permeabilities exceed those predicted by equation (9). An explanation is indicated by the effective pore radii of the investigated samples, which do not decrease as rapidly with decreasing porosity as suggested by equation (8) for the effective radius. For the medium range of porosities $0.01 < \phi < 0.1$ we can improve the permeability estimates by assigning a fixed value to the effective hydraulic pore radius in the Kozeny-Carman equation, equation (1):

$$r_{\text{eff}} = r_{\text{eff, min}} = 200 \text{ nm}. \text{ This is equal or near the mean of values for the effective hydraulic pore radius determined for medium range porosities from equations (1), (2), and (7) from measured permeabilities and porosities. Instead of equation (9) for permeability, a combination of the Kozeny-Carman equation (1), the formation factor, equation (2), and tortuosity, equation (7), yields:}$$

$$k = \phi \frac{r_{\text{eff, min}}^2}{8 \times 0.67} = 7463 \phi^2 \quad \text{(in nm}^2\text{)}. \tag{10}$$

For very low porosities $< 0.01$ this expression still yields permeabilities that are too small compared to the measured data. The values for the effective hydraulic pore radius determined for this range of porosities from our data and porosities from equations (1), (2), and (7) are clearly smaller than 200 nm, but not smaller than 50 nm. For this range it therefore makes sense to assume a fixed minimum value $r_{\text{eff, min}} = 50 \text{ nm}$ and a fixed maximum tortuosity $T_{\text{max}} = 10$ (Pape et al., 1987b). According to equation (7) for tortuosity, equation (10) for permeability then reads:

$$k = \phi \frac{r_{\text{eff, min}}^2}{8} T_{\text{max}} = 31 \phi \quad \text{(in nm}^2\text{)}. \tag{11}$$

The sum of the expressions for permeability in equations (9), (10), and (11) provides an approximation of the permeability over the whole range of porosity studied:

$$k = 31 \phi + 7463 \phi^2 + 191(10 \phi)^{10} \quad \text{(in nm}^2\text{)}. \tag{12}$$

The linear combination of the expressions for the low, medium, and high porosity ranges is permissible since for a given porosity, the expressions for the other two porosity ranges do not contribute significantly, due to the difference in powers of porosity. The third term in the average permeability-porosity relationship, equation (12), characterizes the fractal behaviour of sandstones, and the power of 10 reflects the fractal dimension $D=2.36$. Deviations from this value of the fractal dimension result in powers other than $-0.25$ in equation (6) for porosity. This results in a correspondingly modified exponent for the third term in the average permeability-porosity relationship, equation (12). A fractal dimension $2< D < 2.36$ corresponds to an exponent between 3 and 10, and a fractal dimension $D > 2.36$ to an exponent greater than 10. The $k$-$\phi$ relationship (equation 12) is of the general type $k = A\phi + B\phi^3 + C(10\phi)^{10}$, where the factors $A$, $B$, and $C$ in equation (12) are valid for an average type of sandstone. When applied to a specific basin, it needs to be calibrated. With respect to the Rotliegend data set the average $k$-$\phi$ relationship, equation (12), tends to underestimate the measured permeability, since the investigated northeast German Rotliegend sandstones are characterized by a relatively large permeability at any given porosity. The coefficients of $\phi$, $\phi^3$, and $\phi^{10}$ in the average $k$-$\phi$ relationship (equation 12) are calibrated to the Rotliegend data set by nonlinear regression. Minimizing the least-squares standard deviation of permeability calculated from $k = A\phi + B\phi^3 + C(10\phi)^{10}$ yields the following expression:

$$k = 155\phi + 37315\phi^2 + 630(10\phi)^{10} \tag{13}$$

Example

For practical applications the fractal approach allows one to obtain permeability profiles from conventional industry porosity logs. We apply the average $k$-$\phi$ relationship, equation (12), to porosity values (Figure 3) which are based on the combined information from acoustic, density, and neutron logs. The logs were recorded in a borehole in the Rotliegend formation in northwest Germany. Most part of the section shown was cored. Porosity and permeability were determined in the laboratory and are available for calibration. Core permeability is well correlated with permeability calculated from core porosity and the average $k$-$\phi$ relationship,
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equation (12), This is expressed by \( R^2 = 0.81 \), where \( R \) is the correlation coefficient. Therefore the average \( k-\phi \) relationship, equation (12), appears to be adequate to calculate a continuous permeability log for this basin from log porosity, without any further calibration. The result is in good agreement with the individual permeability values measured on core (Figure 3).

Conclusions

In the classical capillary-bundle model of Kozeny and Carman, \( r_{\text{eff}}, \gamma, \text{and } \phi \) are independent parameters. In fact, this is true for general porous media. However, in rocks which have experienced the same sedimentation and diagenesis history, these parameters become correlated as in the case of the analyzed Rotliegend sandstone samples. Based on a fractal model for porous rocks, new relations were established in which effective radius, tortuosity and porosity are connected through the fractal dimension \( D \). A default value of \( D = 2.36 \) was shown to be useful for the interpretation of data from an ensemble of Rotliegend sandstone samples. Permeability can be estimated from a power-law relation between permeability and porosity. This fractal model is flexible and applicable over a wide range of porosities, and it can be adjusted to different rock types. The average fractal permeability-porosity relationship was successfully calibrated to a comprehensive data set measured on Rotliegend sandstone samples. A group of variants of this correspond to different types of sandstones ranging from clean to shaly. When applying this relationship to a specific basin, however, its coefficients need to be determined anew, based on core data from this specific basin.

In conclusion, the fractal approach provides a simple and versatile technique that can be easily used to derive permeability from porosity determined from conventional industry porosity logs.

References


Figure 3: Left: Porosity measured on cores (triangles) and derived from logs (line); Right: Permeability measured on cores (triangles) and derived from the average \( k-\phi \) relationship, equation (12), and the porosity log on the left (line); The borehole section consists mainly of sandstone with intercalated shale (hatchure).

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